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Technical Note

Aerosol particle transport in a natural convection flow onto a vertical flat plate

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Abstract

An approach based on the theory of high-mass transfer rates with a blowing parameter, which is extended from previously published correlations, is developed for evaluating aerosol particle deposition onto a vertical flat plate. The airflow driven by a buoyancy force is considered as a two-dimensional, incompressible and steady-state laminar natural convective flow. The approach can be easily implemented in a hand calculation for making predictions of the particle deposition rates under a room environment. While the predicted results are compared with data from previous works using similarity analysis and through numerical integration, we find that the comparison shows a very good agreement.

1. Introduction

Radiometric force by a temperature gradient (thermophoresis) that enhances small particles moving toward cold surfaces and away from hot surfaces is of environmental interest. The significant role of thermophoresis upon particle deposition onto a surface in laminar boundary layer flows is now well understood theoretically. Goren [1] developed the thermophoretic deposition of particles in a laminar compressible boundary layer flow past a flat plate. Some other works [2–6] proposed models for particle deposition in coupled thermophoresis and Brownian diffusion. Gokoglu and Rosner [7] and Tsai [8] used the concept of high-mass transfer rate and reported correlations for predicting the particle deposition rates in a laminar forced convection with the presence of thermophoresis.

Here, we are interested in the problem of aerosol particle deposition onto a vertical surface, which is of importance for indoor air quality and nuclear reaction safety. The predicted results are quite useful for estimating the rates of precipitation for dust, soot and mist from the atmosphere. Mills and Wassel [9] and Nazaroff and Cass [10] used a similarity transformation to obtain the deposition rates due to the coupling of thermophoresis and natural convection. Tsai and Lin [11] proposed an approach through numerical integration for evaluating the particle deposition rates. In this study, we developed a simple approach, which was extended from previously published thermophoretic particle transport correlations [7,8], to predict particle deposition rates in natural convection flows. The predicted results from this approach were checked for accuracy against the numerical calculations for a self-similar boundary layer flow.

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2. Similarity analysis

For this natural convection flow over a vertical flat plate, the coordinates were x measured along the surface and y perpendicular to the system. The corresponding velocity components were u and v, respectively. The vertical plate surface was maintained at a temperature, T_w, and the ambient air was at a different temperature, T_e, in which $T_e > T_w$ for a cold surface and $T_e < T_w$ for a hot surface. We assumed the particle concentration to be dilute and zero at the wall. Thus, for a steady laminar flow, the governing conservation equations with the Boussinesq approximation are

$$\text{Mass:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\text{Momentum:} \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_g \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_e), \tag{2}$$

$$\text{Energy:} \quad u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$\text{Particle:} \quad u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = D\frac{\partial^2 N}{\partial y^2} - \frac{\partial}{\partial y}(V_T N), \tag{4}$$

and the boundary conditions at $y = 0$ and $y \to \infty$ are

$$\begin{aligned} &y = 0,\ u = v = 0,\ T = T_w,\ N = 0; \\ &y \to \infty,\ u = 0,\ T = T_e,\ N = N_e, \end{aligned} \tag{5}$$

where β is the coefficient of the fluid thermal volumetric expansion ($\beta = 1/T$ and here $T = T_e$ for an ideal gas), N is the particle concentration, and V_T the thermophoretic velocity recommended by Talbot et al. [12]

$$V_T = -\kappa\nu_g \frac{\nabla T}{T} = -\kappa\nu_g \frac{1}{T}\frac{\partial T}{\partial y}. \tag{6}$$

The $\kappa\nu_g$ value represents the thermophoretic diffusivity, where κ is the thermophoretic coefficient that is a function of the particle size and materials (see [4] for a suggestion) and ν_g is the air kinematic viscosity. A representative value for particles smaller than 1 μm is 0.5.

The governing equations had similar forms and can be described using a dimensionless stream function, $f(\eta)$, a dimensionless temperature, $\theta(\eta)$, and a dimensionless particle concentration $\phi(\eta)$ defined as

$$f(\eta) = -\frac{\psi}{c\nu_g x^{3/4}}; \quad \theta(\eta) = \frac{T - T_e}{T_w - T_e} = \frac{T - T_e}{\Delta T}; \quad \phi(\eta) = \frac{N}{N_e}, \tag{7}$$

where $\eta = cyx^{-1/4}$ and $c = (g\beta|\Delta T|/\nu_g^2)^{1/4}$. Eqs. (1)–(5) after the similarity transformation for f, θ and ϕ are

$$f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + \theta = 0, \tag{8}$$

$$\frac{1}{Pr}\theta'' + \frac{3}{4}f'\theta' = 0, \tag{9}$$

$$\frac{1}{Sc}\phi'' + \left[\frac{3}{4}f + \left(\kappa\frac{\Delta T}{T}\right)\theta'\right]\phi' + \left[\left(\kappa\frac{\Delta T}{T}\right)\left(\theta'' - \frac{\Delta T}{T}\theta'2\right)\right]\phi = 0 \tag{10}$$

And boundary conditions are

$$\begin{aligned} &f(0) = f'(0) = 0, \quad f'(\infty) = 0, \\ &\theta(0) = 1, \quad \theta(\infty) = 0, \\ &\phi(0) = 0, \quad \phi(\infty) = 1, \end{aligned} \tag{11}$$

where Pr is the Prandtl number. Solutions for $f(\eta)$ and $g(\eta)$ from the above equations for air ($Pr = 0.72$ used) can be obtained using the methods of quasilinearization and finite differences.

3. Approach for particle deposition velocity

Particle flux to the wall surface can be determined using the definition

$$J_s = -D\frac{\partial N}{\partial y}\bigg|_{y=0} = -D\phi'(0)N_e c x^{-1/4}. \tag{12}$$

The deposition velocity is customarily defined as the particle flux divided by the free stream concentration,

$$V_d = \frac{J_s}{N_e} = -D\phi'(0)cx^{-1/4}. \tag{13}$$

The particle deposition velocity for aerosols with a fixed x and ΔT is a function of the slope of the concentration profile at the wall surface $\phi'(0)$. There are two ways to determine the value of $\phi'(0)$. One is a direct method from a similar solution for concentration profile [10], and the other is from the asymptotic approach as well as numerical integration [11]. In this study, an alternative approach, which easily can be estimated and does not need numerical schemes, would be developed.

From Eqs. (12) and (13), the ratio of particle deposition fluxes between an existing thermophoretic effect and the limiting case of the zero thermophoretic velocity is

$$\frac{J}{J^*} = \frac{V_\mathrm{d}}{V_\mathrm{d}^*} = \frac{\phi'(0)}{\phi'(0)|_{V_\mathrm{T}=0}} = F_\mathrm{T}, \tag{14}$$

where the asterisk denotes values in the limiting case for $V_\mathrm{T} = 0$, and F_T is called a thermophoretic factor. By applying a Couette flow analysis, Gokoglu and Rosner [7] introduced that F_T can be expressed as the production of factors due to thermophoretic blowing (or suction) and source (or sink)

$$F_\mathrm{T} = \left(\frac{B_\mathrm{m}}{\exp B_\mathrm{m} - 1}\right)\left(\frac{N_\mathrm{m}}{N_\mathrm{e}}\right), \tag{15}$$

where B_m is called a blowing parameter and negative for suction. N_m is the particle concentration near the plate surface and the thermophoretic source for air can be estimated as $N_\mathrm{m}/N_\mathrm{e} \approx \exp(-\kappa\Delta T/T)$ [7]. Under a room environment where $|\Delta T|$ is often less than 30 K, the effect of the thermophoretic source (or sink) nears unity; thus, $N_\mathrm{m}/N_\mathrm{e} = 1$ considered in this study.

Usually, the particle Schmidt number for aerosols is very large ($\geq 10^3$), and the resulting concentration boundary layer thickness, δ_m, is much thinner than the hydrodynamic and thermal boundary layers, δ and δ_t. The particle deposition flux due to thermophoretic blowing may be approximated using

$$J_{\mathrm{s,T}} \cong \lim_{y \to \delta_\mathrm{m}} (V_\mathrm{T} N) = V_{\mathrm{T,m}} N_\mathrm{m}. \tag{16}$$

The blowing parameter is defined as [13,14]

$$B_\mathrm{m} = \frac{V_{\mathrm{T,m}}\delta_\mathrm{m}}{D} = \frac{V_{\mathrm{T,m}}}{V_\mathrm{d}^*}, \tag{17}$$

where V_d^* is the particle deposition velocity in the limiting case of zero thermophoretic velocity, and the thermophoretic velocity $V_{\mathrm{T,m}}$ near the plate surface can be regarded as a blowing-like velocity,

$$V_{\mathrm{T,m}} = -\kappa \frac{v_\mathrm{g}}{T}\frac{\partial T}{\partial y}\bigg|_{y \to \delta_\mathrm{m}} \approx -\kappa v_\mathrm{g}\frac{\Delta T}{T}\theta'(0)cx^{-1/4}. \tag{18}$$

The resulting particle deposition velocity with the assumption of $N_\mathrm{m}/N_\mathrm{e} = 1$ is

$$V_\mathrm{d} = V_\mathrm{d}^*\left(\frac{B_\mathrm{m}}{\exp B_\mathrm{m} - 1}\right). \tag{19}$$

This equation indicates that the limiting case in V_d^* must be known if we want to determine the particle deposition velocity and the blowing parameter. Commonly, in the mass transfer analysis, the V_d^* quantities obtained are from the analogy between heat and mass transfer. However, since the flow field, Eq. (2), is coupled with the temperature but independent of the particle concentration, this analogy is not a suitable model for the natural convection. To get a correct value of V_d^*, we developed an empirical equation using the order of magnitude method. From the scale analysis for boundary flows, due to the uncoupling between the flow and concentration fields, the ratio δ_m/δ is of $O(Sc^{-1/3})$ (same as $\delta_\mathrm{T}/\delta \sim O(Pr^{-1/3})$ in forced convection) and δ is of $O(cx^{-1/4})$ (in the laminar natural convection). Thus, we treated V_d^* as

$$V_d^* = \frac{D}{\delta_\mathrm{m}} = ADcx^{-1/4}Sc^{1/3}, \tag{20}$$

where A is a constant and correlated to be 0.55 from

Table 1
A comparison of $V_\mathrm{d} \times 10^4$ cm/s given by predicted results and numerical results from Nazaroff and Cass [10] and Tsai and Lin [11]

d_p (μm)	$T_\mathrm{w} - T_\mathrm{e}$ (K)									
	−10	−4	−2	−1	−0.1	0.1	1	2	4	10
0.01										
Nazaroff and Cass [10]	47.3	32.6	25.7	21.0	11.5	11.5	19.9	23.0	25.8	26.9
Tsai and Lin [11]	48.5	33.2	26.4	21.6	11.8	11.8	20.5	23.7	26.6	28.0
Predicted results	47.0	32.6	26.2	21.5	11.8	11.8	20.5	23.7	26.8	28.9
0.1										
Nazaroff and Cass [10]	18.0	6.1	3.0	1.8	0.68	0.62	0.66	0.39	0.07	–
Tsai and Lin [11]	18.3	6.2	3.1	1.8	0.68	0.62	0.66	0.39	0.07	–
Predicted results	18.0	5.9	2.9	1.7	0.68	0.62	0.72	0.49	0.17	–
1.0										
Nazaroff and Cass [10]	17.9	5.7	2.4	1.0	0.11	0.05	–	–	–	–
Tsai and Lin [11]	18.0	5.8	2.4	1.0	0.11	0.55	–	–	–	–
Predicted results	18.0	5.7	2.4	1.0	0.11	0.05	–	–	–	–

the solutions of similar boundary layer flows. Substitution of Eqs. (18) and (20) into Eq. (17) gives the blowing parameter as

$$B_\mathrm{m} = \frac{-\kappa v_\mathrm{g}(\Delta T/T)\theta'(0)}{0.55 D Sc^{1/3}}. \tag{21}$$

Hence, from Eqs. (20) and (21), the particle deposition velocity in Eq. (19) can be easily calculated ($\theta'(0) = -0.3567$ [15]).

4. Conclusion and discussion

In the mass transfer textbooks [13,14], a theory of high-mass transfer rates with blowing parameters is useful for determining particle deposition rates and confirmed for forced convection with thermophoretic effects in Refs. [7,8]. In this study, a simple approach for particle deposition onto a vertical flat plate based on that theory was extended and applied in a natural convection flow. The prediction deposition velocity in a room environment is

$$V_\mathrm{d} = V_\mathrm{d}^*\left(\frac{B_\mathrm{m}}{\exp B_\mathrm{m} - 1}\right),$$

where V_d^* and B_m can be obtained from Eqs. (20) and (21), respectively. To examine the prediction results, particles of 0.01, 0.1, and 1.0 μm corresponding to the value of the Schmidt number 2.87×10^2, 2.22×10^4, and 5.42×10^5 were selected [16]. Table 1 is a comparison of prediction deposition velocities for $x = 1$ m, $T_\mathrm{e} = 293$ K and $\kappa = 0.5$ with the numerical results from similarity solutions [10] and thorough numerical integration [11]. The table shows that the agreement is very good in which the maximum possible error is less than 3% or 0.1×10^{-4} cm/s.

Usually, the engineer requires the total or average particle transport from a surface and is not too interested in the variation in particle flux along the surface. For this purpose, this approach is not only an easier way for determining the local particle deposition flux but also the average particle deposition flux using simple integration calculus,

$$\bar{J}_\mathrm{s} = \frac{1}{L}\int_0^L J_\mathrm{s}\,\mathrm{d}x = \frac{N_\mathrm{e}}{L}\int_0^L V_\mathrm{d}\,\mathrm{d}x = 0.733 D Sc^{1/3} c L^{-1/4}\left(\frac{B_\mathrm{m}}{\exp B_\mathrm{m} - 1}\right), \tag{22}$$

where L is the plate length.

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